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Modeling the Planetary Boundary Layer: Frictional Influence

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1.0 Introduction

The modeling of the layer of frictional influence in the NMC models has not been subject to systematic study. There is some evidence that the presently used formulation possesses deficiencies. In this paper, we review the simplest ideas for parameterization of a boundary layer of frictional influence and note some aspects of the theoretical weaknesses of the current formulation. During the current fiscal year, experimental computations with revised versions of the boundary layer physics will be undertaken. Results of those experiments will be published.

2.0 The Structure of the Planetary Boundary Layer

The level at which the mean wind vanishes is usually defined as the *roughness length* and denoted by z_o. This quantity is usually estimated at 3% of the obstacle heights. Below z_o one assumes that molecular exchange is dominant.

Between z_o=z and h=z (h≃50 to 100m), eddy flux of momentum is relatively very large compared to its divergence. Thus, the law for momentum conservation,

$$\frac{d\tilde{v}}{dt} + f\,\tilde{k} \times \tilde{v} + \alpha\,\nabla p = \frac{\partial}{\partial z}\left[K\,\frac{\partial \tilde{v}}{\partial z}\right] \qquad (1)$$

is approximated by the statement

$$\frac{\partial}{\partial z}\left[K\,\frac{\partial \tilde{v}}{\partial z}\right] \simeq 0 \qquad (2)$$

The quantity $K\frac{\partial \tilde{v}}{\partial z}$ is the eddy flux of momentum and is related to the eddy stress $\vec{\tau}$ by

$$-\alpha\,\frac{\partial \vec{\tau}}{\partial z} = \frac{\partial}{\partial z}\left[K\,\frac{\partial \tilde{v}}{\partial z}\right] \qquad (3)$$

Variations in α are usually neglected, to write

$$\vec{\tau} \simeq -\rho K\,\frac{\partial \tilde{v}}{\partial z} \qquad (4)$$

The eddy viscosity coefficient K has the dimensions, $cm^2 sec^{-1}$, in cgs units.

Based on G. I. Taylor's meteorological application of Prandtl's concept of a mixing length, ℓ, the quantity K is dimensionally related to the wind shear by

$$K = \ell^2 \frac{\partial |\tilde{v}|}{\partial z} \tag{5}$$

where $|\tilde{v}|$ is the magnitude of the vector wind. The vector $\tilde{v}$ does not vary in direction within the region near the ground; thus

$$\left|\frac{\partial \tilde{v}}{\partial z}\right| = \frac{\partial |\tilde{v}|}{\partial z} \tag{6}$$

Finally, the mixing length ℓ is argued to be proportional to distance above the ground

$$\ell = kz \tag{7}$$

the k is a universal constant (von Karman's). Therefore,

$$K = (kz)^2 \left|\frac{\partial \tilde{v}}{\partial z}\right| \tag{8}$$

When (8) is used in Eq. (2), one gets

$$kz \frac{\partial u}{\partial z} = \sqrt{\frac{|\vec{\tau}_o|}{\rho}} \equiv u_* \tag{9}$$

in which $\vec{\tau}_o$ is the stress at the ground and we have written u for the wind speed. The vectoral character of Eq. (2) is retained implicitly. The quantity, u_*, is the *friction velocity*; one has

$$u_*^2 = \frac{|\vec{\tau}_o|}{\rho} \tag{10}$$

Integration of Eq. (9) gives

$$u = \frac{u_*}{k} \ell n\left(\frac{z}{z_o}\right) \tag{11}$$

the logarithmic profile.

2.1 Diabatic Effects

The preceding development did not take into account the possible influence of buoyancy arising from the thermal stratification of the atmosphere near the ground. Actually, the lapse-rate of temperature in this region undergoes very large variations. A parameter which is useful in quantifying the relative significance of buoyancy as against wind shear is the Richardson's number, Ri,

$$Ri = \frac{g}{\theta} \frac{\partial \theta}{\partial z} \Big/ \left(\frac{\partial u}{\partial z}\right)^2 \qquad (12)$$

It is common practice to delineate three regimes:

$$\begin{array}{ll} \text{Free convection} & Ri \leq -.03 \\ \text{Forced convection} & -.03 < Ri < 0.5 \\ \text{Stable} & Ri \geq 0.5 \end{array} \qquad (13)$$

in which the Ri values are appropriate for estimates made at about 4 m.

The value of $|Ri|$ usually increases with height so the values given to delineate the range are variable depending upon the level of estimation of Ri.

In the domain of forced convection, the wind profile suggested by the Monin-Obukhov dimensional analysis is

$$u = \frac{u_*}{k} \ln\left(\frac{z}{z_o}\right) - \frac{Q}{c_p \rho\ u_*^2} \frac{\beta g}{\overline{\theta}} (z - z_o) \qquad (14)$$

the so-called log-linear profile. The parameter Q is the heat flux counted positive when it is directed upwards (lapse conditions). β is an empirical constant, value about 2, the other symbols are:

c_p	specific heat at constant pressure
ρ	air density
g	gravity acceleration
$\overline{\theta}$	mean potential temperature

In deriving (14) we have used the eddy viscosity coefficient formulation,

$$K = [kz(1 - \beta Ri)]^2 \left|\frac{\partial \tilde{v}}{\partial z}\right| \tag{15}$$

Thus, by reference to the mixing length Eq. (5), we have

$$\ell = kz(1 - \beta Ri). \tag{16}$$

So, lapse stratification is modeled as increasing the mean free path or mixing length, and stable stratification as decreasing that length.

This formulation of the forced convection regime is widely accepted. The majority of naturally occurring situations are well treated by it.

However, the formulation of the free convection and stable regimes is not so widely agreed upon. Observational data is more difficult to come by and in the stable regime little turbulence occurs.

Both the GFDL* and NMC formulations of the boundary layer are appropriate under neutral conditions, Ri = 0, only.

2.2 Drag Coefficients

The stress, $\vec{\tau}$, acting on a surface may be related to a wind measurement at a given altitude, say h, by use of a parameter called the *drag coefficient*. One may write,

$$\vec{\tau}_o = - \rho\ C_D |\tilde{v}(h)| \tilde{v}(h) \tag{17}$$

in which C_D is dimensionless. Since the stress vector is assumed to be anti-parallel to the direction of $\tilde{v}$ in (17), it follows that $\tilde{v}$ must be measured within the layer of constant flux, i.e., within the Prandtl layer.

It is clear from Eq. (10) and Eq. (17) that one may write

$$u_*^2 = C_D |\tilde{v}(h)|^2 \tag{18}$$

If we write $\quad u(h) = |\tilde{v}(h)| \quad$ then

$$C_D \doteq \left[\frac{u_*}{u(h)}\right]^2 \tag{19}$$

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*cf. Miyakoda, et al., 1969, and Smagorinsky, et al. 1965.

So from Eq. (14) one may derive

$$u(h) = u_*[\frac{1}{k}\ \ln\frac{h}{z_o} - \frac{Q}{c_p\rho\ u_*{}^3}\ \frac{\beta g}{\theta}(h-z_o)] \tag{20}$$

thence it follows that

$$C_D = [\ \frac{1}{k}\ \ln\frac{h}{z_o} - \frac{Q}{c_p\rho\ u_*{}^3}\ \frac{\beta g}{\theta}(h-z_o)\,]^{-2} \tag{21}$$

In the adiabatic case, Q=0, one has

$$C_{D_{ad.}} = k^2/(\ln\frac{h}{z_o})^2 \tag{22}$$

One may therefore expect to be able to specify the value of C_D in neutral conditions simply by knowing the roughness height; the parameter C_D must be chosen appropriately for the height at which the wind is estimated.*

In diabatic stratification, we see that the formulation of the drag coefficient is more complex.

2.3 Geostrophic Drag Coefficient

The theory of the Prandtl layer has been developed largely from the viewpoint of micrometeorology. From the macroscopic viewpoint--that appropriate to the global model--one would prefer a less elaborate formulation of the stress acting at the Earth surface.

To that end, one may appreciate the work of Lettau (1959) and Blackadar (1963) which is based on estimates of the stress using the surface geostrophic wind. In a boundary layer model developed by the present writer (Gerrity, 1967), the geostrophic theory was utilized with considerable success.

The surface Rossby number is defined by Lettau as

$$R\sigma_o \equiv G/(fz_o) \tag{23}$$

in which G is the surface geostrophic wind speed and f is the Coriolis

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*cf. p. 21 in Priestly, C.H.B., Turbulent Transfer in the Lower Atmosphere, Univ. of Chicago Press, Chicago, 1959, 130 + xii.

parameter. He then defines the *geostrophic drag coefficient* as

$$C_g = \frac{u_*}{G} \tag{24}$$

Lettau provides data on the relations between u_* and G in neutral conditions; we used later data compilations by Blackadar to get

$$u_{*_N} = G[.07625 - .00625 \log_{10} R_{\sigma_o}] \tag{25}$$

the quantity in brackets is the geostrophic drag coefficient.

Lettau provides an indication that the value of C_g in diabatic conditions [within the forced convective regime] is about 20% greater in lapse and 20% less in stable conditions.

Now if one accepts (25), it is easy to derive the wind speed at any level within the neutral Prandtl layer. To determine the vector wind we need a relation between the direction of the surface geostrophic wind and actual wind direction. Since the wind is always directed toward lower pressure [cf. VI in Charney and Eliassen, 1949] in the frictional layer, we need only specify the angle ψ between the two vectors: geostrophic wind and real wind.

Based on data presented by Blackadar (1963) we found that

$$\psi = a(\log_{10} R_{\sigma_o}) + b \log_{10} R_{\sigma_o} + c \tag{26}$$

with a = .625, b = - 12.75, and c = 80.625.

So ψ varies between 32.5^o for $R_{\sigma_o} = 10^5$ and 15.6^o for $R_{\sigma_o} = 10^{10}$.

The use of these relationships will permit the calculation of the information usually gleaned from the Prandtl layer. It is my opinion that an information layer at the top of the Prandtl layer is not required for reasonable accuracy in modeling the stress near the ground. But a consistent use of the geostrophic approach should be employed.

It should be noted that Lettau's geostrophic drag coefficient (cf. Eq. 24) is related to the surface stress $\vec{\tau}_o$ by the equation,

$$u_*^2 = (GC_g)^2 = \frac{|\vec{\tau}_o|}{\rho} \tag{27}$$

But if this is compared with Eq. (17),

$$C_D|\tilde{v}(h)||\tilde{v}(h)| = \frac{|\tilde{\tau}_o|}{\rho}$$

it will be seen that a difference occurs in the *exponentiation* of C_g as against C_D. This difference is very important when one comes to quoted values of the *drag coefficient*. If one were to assume Lettau's values which are

$$C_g \simeq 0.03$$

then the value of C_D is

$$(.030)^2G^2 = C_D|\tilde{v}(h)||\tilde{v}(h)|$$

or

$$C_D = .9 \times 10^{-3}[G^2/|\tilde{v}(h)|^2] \tag{28}$$

We shall return to this point in section 5.

2.4 The Ekman Layer

Above the Prandtl layer is a layer of transition within which the surface frictional influence is absorbed. This layer is usually called the Ekman layer.

Rossby and Montgomery (1935) suggest that within this region, the eddy viscosity diminishes to zero linearly with height. This is the formulation adopted by GFDL. The layer is variously treated as having a depth between 1 and 2.5 km; the latter is used by GFDL.

Ekman's theoretical model was derived for the ocean in a barotropic steady state. Blackadar has recently shown the significance of the baroclinicity of this layer, and it is obvious that the steady state hypothesis is unacceptable except as a first approximation.

2.5 The Equatorial Boundary Layer

Reflection on the structure of the wind profile in the Southern Hemisphere suggests that near the equator the boundary layer must be treated quite differently.

It is clear that frictional effects will still be important but the absence of geostrophic controls on the wind field will require some additional thought. Some insight may be gained by a study of wind observations in equatorial latitudes.

3.0 Critique of the Current Model

At present the NMC model uses a 50 mb deep layer in which the effects of friction are modeled. If δ is the thickness of the boundary layer, one has from the hydrostatic equation:

$$\delta = \frac{R\ T_v}{g(p_g - 25)} \cdot 50 = 1.448 \times 10^5 \frac{T_v}{p_g - 25} \quad \text{cm}$$

in which p_g is the station pressure in mb, T_v is the mean virtual temperature of the layer, R is the gas constant for dry air (2.8704×10^6 cgs), and g is gravity acceleration (980 cm sec^{-2}).

	δ m	T_v(°K) 260	280	300
p_g(mb)	1025	376	405	434
	925	414	441	477
	825	470	506	542

Table 1. Depth of NMC Boundary Layer as a function of temperature and surface pressure.

Table 1 above indicates a variation in the geometric depth of the NMC boundary layer. It is deeper at high altitude stations and at warmer temperatures.

If we consider the formula (Eq. 22) for the adiabatic drag coefficients dependence on the altitude of measurement, it is clear that one ought to expect a decrease in C_D for higher altitudes and warmer temperatures. The current formulation is the reverse of this, since the drag coefficient is generally larger over mountainous grid points.

The NMC values of C_D are taken from the work of Cressman (1960), who was attempting to formulate a frictional influence for use with the barotropic model. Cressman's drag coefficient formulation incorporates a parameterization of the influence of mountain wave momentum transfer which is not included in the usual boundary layer analysis.

Cressman defines the Drag Coefficient [his Eq. (7)] with V_h the wind at the *gradient level*

$$C_c V_h^2 = \frac{|\vec{\tau}|}{\rho} \tag{29}$$

He then asserts that C_c may be thought of as a linear combination of two physical factors. He writes C_2 for the drag associated with the form of the relief and C_1 for a *relatively constant* value associated with the drag over flat land or the oceans.

To C_1, Cressman ascribes an average value of 0.12×10^{-2}; he quotes values as follows for C_2,

(a) land with trees and low relief: $C_2 = 0.1$ to 0.2×10^{-2}

(b) moderately high mountains: $C_2 = .2$ to $.5 \times 10^{-2}$

(c) very high mountains: $C_2 = .5$ to $.9 \times 10^{-2}$

He used the C_2 estimates to fix the parameter K in the formula,

$$\frac{Knh}{2} = d\, C_2 \tag{30}$$

in which n is the number of ridges of height h running across a grid square, of side d, perpendicular to the wind. The formula was then applied to topographic charts to produce a map of C_2. He then added the constant value of C_1 to the mapped value of C_2 to obtain a final map of C_c. The average value (area weighted) of C_c is 0.22×10^{-2}, which is comparable to estimates made by other workers cited in Cressman's paper.

It seems clear that in Cressman's work, major emphasis is put on the estimation of the so-called *form drag*. Cressman's estimate of this parameter is based on Sawyer's and Scorer's method for estimating momentum exchange due to internal gravity waves excited by the passage of an air stream over undulating terrain. Although it is possible that this formulation has some utility, I think it is fair to criticize the method on theoretical grounds.

It is not at all clear that the excitation of internal gravity waves is a property of the topography alone. Clearly, if the stratification is neutral, no internal gravity waves are possible. Furthermore, the direction of the wind relative to the ridge line is important; but this formulation does not take that into account. The influence of small regularly spaced ridge lines [the situation in older mountain chains] would seem more significant in exciting wave motion than the influence of random rugged peaks, as in the Rocky or Himalayan Mountains.

3.1 The Equation Governing Flow Near the Ground

In σ-coordinates, the equation governing horizontal motion is (neglect map factor)

$$\frac{\partial \tilde{v}}{\partial t} + (\tilde{v}\cdot\nabla)\tilde{v} + \dot{\sigma}\frac{\partial \tilde{v}}{\partial \sigma} + f\tilde{k}x\tilde{v} + c_p\theta\tilde{\nabla}\pi + \tilde{\nabla}\phi = + g\frac{\partial \sigma}{\partial p}\frac{\partial \vec{\tau}}{\partial \sigma} \tag{31}$$

If the depth of the boundary layer is such that p_σ is a constant, one has the continuity equation

$$\tilde{\nabla}\cdot\tilde{v} + \frac{\partial \dot{\sigma}}{\partial \sigma} = 0 \tag{32}$$

So Eq. 31 may be written,

$$\frac{\partial \tilde{v}}{\partial t} + \vec{i}\ \tilde{\nabla}\cdot u\tilde{v} + \vec{j}\ \tilde{\nabla}\cdot v\tilde{v} + \frac{\partial}{\partial \sigma}\dot{\sigma}\tilde{v} + f\tilde{k}x\tilde{v} + c_p\theta\tilde{\nabla}\pi + \tilde{\nabla}\phi = + g\frac{\partial \sigma}{\partial p}\frac{\partial \vec{\tau}}{\partial \sigma} \tag{33}$$

in which $\vec{i}$ is the unit vector in the x-direction and $\vec{j}$ the unit vector in the y-direction. Suppose that Eq. (33) is vertically integrated between $\sigma = 0$ and $\sigma = 1$ with the boundary condition $\dot{\sigma} = 0$ at $\sigma = 1$. Then if a bar over a quantity denotes a vertical average, one has*

$$\frac{\partial \bar{\tilde{v}}}{\partial t} + \vec{i}\ \tilde{\nabla}\cdot\overline{u\tilde{v}} + \vec{j}\ \tilde{\nabla}\cdot\overline{v\tilde{v}} - (\dot{\sigma}\tilde{v})_o + \overline{f\tilde{k}x\tilde{v}} + \overline{c_p\theta\tilde{\nabla}\pi} + \overline{\tilde{\nabla}\phi} = - g\frac{\partial \sigma}{\partial p}(\vec{\tau}_0 - \vec{\tau}_1) \tag{34}$$

*N.B. The $\vec{\tau}_o$ means $\vec{\tau}$ at $\sigma = 0$, the top of the boundary layer.

If $\nabla\theta$ is small, we may write

$$c_p \overline{\theta \tilde{\nabla} \pi} = c_p \tilde{\nabla} \overline{T} \tag{35}$$

Let us also define $\overline{u\tilde{v}} = \vec{T}_x$, $\overline{v\tilde{v}} = \vec{T}_y$. Then Eq. (34) becomes

$$\frac{\partial \bar{\tilde{v}}}{\partial t} + \vec{i}\ \tilde{\nabla}\cdot\vec{T}_x + \vec{j}\ \tilde{\nabla}\cdot\vec{T}_y - (\dot{\sigma}\tilde{v})_o + f\tilde{k}x\bar{\tilde{v}} + \tilde{\nabla}(c_p\overline{T} + \overline{\phi}) = -g\frac{\partial\sigma}{\partial p}(\vec{\tau}_0 - \vec{\tau}_1) \tag{36}$$

If we form the scalar product of $\bar{\tilde{v}}$ with Eq. (36)

$$\bar{\tilde{v}}\cdot\frac{\partial \bar{\tilde{v}}}{\partial t} + \overline{u}\ \tilde{\nabla}\cdot\vec{T}_x + \overline{v}\ \tilde{\nabla}\cdot\vec{T}_y - \dot{\sigma}_o(\bar{\tilde{v}}\cdot\tilde{v}_o) + \bar{\tilde{v}}\cdot\tilde{\nabla}(c_p\overline{T} + \overline{\phi})$$

$$= -g\frac{\partial\sigma}{\partial p}\,[\bar{\tilde{v}}\cdot\vec{\tau}_0 - \bar{\tilde{v}}\cdot\vec{\tau}_1] \tag{37}$$

Now in the NMC model $\vec{\tau}_o$ is zero, i.e., no stress acts on the upper boundary. Also $\bar{\tilde{v}}$ is identified with the wind carried in the middle of the layer and it is *antiparallel* to $\vec{\tau}_1$. Let $\bar{\tilde{v}}\cdot\bar{\tilde{v}} = 2\overline{k}$, with $\overline{k}$ the *kinetic* energy per unit mass:

$$\frac{\partial \overline{k}}{\partial t} + \overline{u}\tilde{\nabla}\cdot\vec{T}_x + \overline{v}\tilde{\nabla}\cdot\vec{T}_y - \dot{\sigma}_o(\bar{\tilde{v}}\cdot\tilde{v}_o) + \bar{\tilde{v}}\cdot\nabla(c_p\overline{T} + \overline{\phi}) = +g\frac{\partial\sigma}{\partial p}(\bar{\tilde{v}}\cdot\overline{\tau}_1) \tag{38}$$

Now $\quad \vec{\tau}_1 = -\rho\ C_c|\bar{\tilde{v}}|\bar{\tilde{v}}$ (39)

by the NMC formula, so

$$\bar{\tilde{v}}\cdot\vec{\tau}_1 = -2\rho\ C_c|\bar{\tilde{v}}|\overline{k} \tag{40}$$

$$\frac{\partial k}{\partial t} + \overline{u}\ \tilde{\nabla}\cdot\vec{T}_x + \overline{v}\ \tilde{\nabla}\cdot\vec{T}_y - \dot{\sigma}_o(\tilde{v}\cdot\tilde{v}_o) + \bar{\tilde{v}}\cdot\tilde{\nabla}(c_p\overline{T} + \overline{\phi})$$

$$= -2g\rho\frac{\partial\sigma}{\partial p}C_c|\bar{\tilde{v}}|\overline{k} = -E \tag{41}$$

The E represents the rate of dissipation of specific kinetic energy within the boundary layer. The term, $\bar{\tilde{v}}\cdot\bar{\tilde{\nabla}}(c_p\overline{T} + \overline{\phi})$ is a production term provided that

$$\frac{\partial \overline{k}}{\partial t} = -\bar{\tilde{v}}\cdot\bar{\tilde{\nabla}}(c_p\overline{T} + \overline{\phi}) > 0 \tag{42}$$

After Charney and Eliassen (op.cit.) and others, one expects approximate balance between the production and dissipation terms. Thus, one finds that the angle formed by $\overline{\tilde{V}}$ and the gradient of $c_p\overline{T} + \overline{\phi}$ is determined by

$$\cos\psi = \frac{-2g\rho\frac{\partial\sigma}{\partial p}\, C_c\overline{k}}{\left|\tilde{\nabla}(c_p\overline{T}+\overline{\phi})\right|} \tag{43}$$

It should therefore be clear that the use of $\overline{\tilde{v}}$ as representative of both the mean boundary layer wind and as an estimate of the gradient wind as is done in the current NMC model is fundamentally erroneous. Since the vertical transport term, $\dot{\sigma}_0(\overline{\tilde{v}}\cdot\tilde{v}_0)$ is a residual remaining after approximate balance of generation and dissipation, its accurate computation is to be seriously doubted.

4.0 Critique of the GFDL Model Friction Formulation

The GFDL model carries one information level at an altitude of 70 m above the ground. This level is considered to be at the top of the Prandtl layer. The wind profile below 70 m is taken to be logarithmic, i.e., it is assumed that the layer is in neutral stratification for computation of the frictional stress. The drag coefficient is given by Eq. (22) and Z_0 is taken to be a constant 1 cm. I do not know if this formulation has been modified to account for the *form drag* of Cressman.

Above the Prandtl layer it is assumed, following Rossby and Montgomery, that the eddy viscosity coefficient decreases linearly to zero at 2500 m. The model has two and one-half layers in the region between z = 70 and z = 2500 m. Thus the convergence of vertical eddy transfer of momentum is explicitly calculated at three wind information levels, 70 m, 640 m, and 1700 m. Variations in stability do not affect this process, but one must recall the *convective adjustment* and ask if this adjustment modifies the Richardson's number in such a way as to imply that stability modifications are never appropriate for the wind field.

It is my opinion that the use of the Prandtl layer wind field information level is not justified solely for calculation of the stress. A geostrophic drag formulation would be as accurate in my opinion. Thus unless other considerations are involved, one might "save this storage" for other uses.

5.0 Summary

In this note, attention is focused upon certain questionable aspects of the formulation of frictional influences in the NMC models. Two points should be specifically noted:

a. The drag coefficients used in the NMC model are too large, since they reflect the "mountain wave drag" consideration of Cressman's original work.

b. The use of the model's boundary layer wind in the frictional drag estimate is physically incorrect; one should rather be using an estimate of the gradient wind.

It seems appropriate to note these points now, since NMC's newer high resolution models ought to be capable of delineating the real effects of friction upon the observed weather.

In addition a parameterization of the influence of orographically induced gravity waves has recently been developed (Collins, 1976). It may serve to incorporate more accurately this effect in a multilayer model than was possible in Cressman's original work with a barotropic model.

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